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# A Mathematical Method for Visualizing Ptolemy's India in Modern GIS Tools[1]

*Keywords*: Ptolemy; GIS; digital archaeology; history of cartography; ancient India

*Summary*: Ptolemy's *Geography* provides latitudes and longitudes for over 6,000 locations known in his time in the ancient world. Unfortunately, many of the coordinates that were chronicled at that time are known to represent a distorted view of the world. We provide a window into Ptolemy's world by systematically converting the ancient coordinates into their modern equivalents and then loading them into modern GIS tools such as Google Earth. We present our methods of estimating the required adjustments along with an overview of our data flow and an initial application of the methods on the data from Book 7 of Ptolemy's work, covering the Indian subcontinent and adjacent parts of Southeast Asia. By using existing research on locations for which we do know the modern equivalents, we develop a mathematical model for estimating the coordinates of the remaining ones, providing a comprehensive conversion of the ancient data set. The end result and value added by this work is a previously unavailable picture of Ptolemy's *'known world'* developed using the same tools we use to better understand our world today, substantially increasing our ability to understand many aspects of our cultural heritage.

## Introduction

Ptolemy's *Geography* provides coordinates for over 6,000 places in the ancient world[2] along with descriptions and related contextual metadata. Combined with other historical sources such as the *Periplus of the Erythraean Sea* (Schoff 1912), this remarkable cartographic dataset provides an image of how the ancient world looked like, contributes to improved understanding and appreciation of our shared cultural heritage and enables further correlation of other ancient datasets through geospatial association.

Unfortunately, Ptolemy was constrained by the cartographic and information technologies available to him at the time of his work. The voluminous catalog he produced with its degree of detail and accuracy is absolutely impressive, but the misunderstandings of the true shape of the world that it reflects substantially limit its usefulness as a modern geospatial reference. Considerable efforts are needed to compensate for errors and misunderstandings, unlock the wealth of information the book contains and make it more directly accessible in a modern context.

In particular, this applies to India, a unique treasure trove for conventional and digital archaeolo-

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[2] This estimate is based on the translation of Ptolemy's catalog by Stückelberger and Grasshoff (2006). We took their place database file, filtered out records that did not list Ptolemy latitude and longitude as well as those with a duplicate ID and arrived at 6,331 distinct places complete with their coordinates.

gists alike.


Figure 1. This map shows our combined known and unknown locations from Ptolemy's Geography for the West part of India before the Ganges using the triangulation approach. The labels shown are the original Ptolemy names translated into English.

This article reports on our results achieved so far. Figure1-3 provide a visual representation.


Figure 2. This map shows our combined known and unknown locations from Ptolemy's Geography for Taprobane and the South part of India before the Ganges using the trian-gulation approach. The labels shown are the original Ptolemy names translated into English.

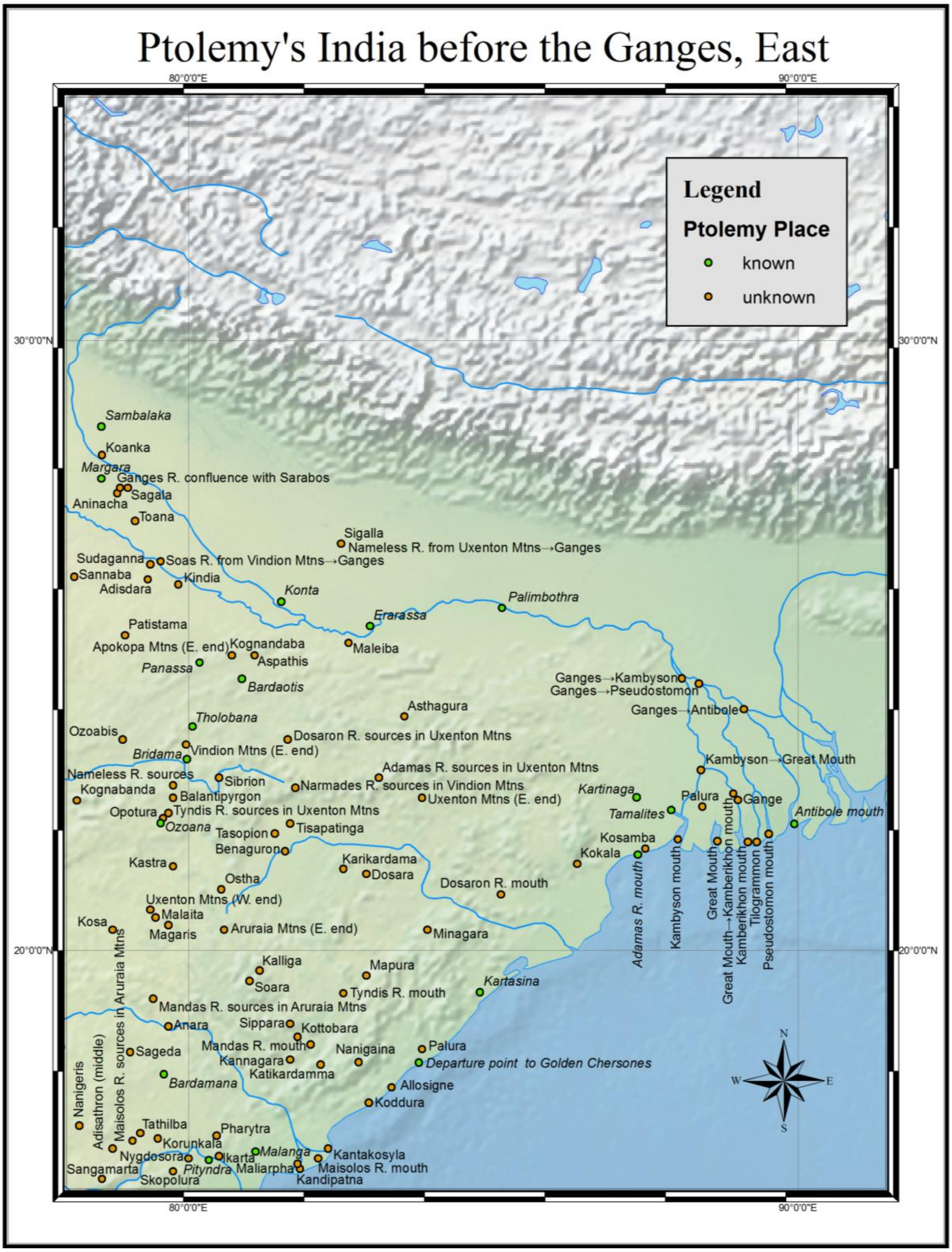


Figure 3. This map shows our combined known and unknown locations from Ptolemy's Geography for the East part of India before the Ganges using the triangulation approach. The labels shown are the original Ptolemy names translated into English.

The work presented here focuses on India, which corresponds to Book 7 of Ptolemy's work, especially Chapters 1 and 4. Our eventual goal is publication of a comprehensive modern version of Ptolemy's catalog that will provide either exact or approximate modern coordinates for every place for which Ptolemy gives us ancient coordinates. By providing such a dataset and corresponding GIS assets, we will enable exploration and visualization of the ancient world in ways that are currently not possible.

This paper is organized as follows. First we review the literature we explored including the surviving translations of Ptolemy's work and their associated commentary. Next we discuss our tools and workflow and how they support our effort. After that we discuss the models we applied to the places for which we do have modern coordinates to predict the coordinates of the others. We next talk about the overall dataset and how we determined where to place the locations that we consider as known. Then we report the results of the effort and provide sample visualizations in modern tools, along with a brief analysis of the relative accuracy of the models applied. Finally, we present our conclusions and discuss potential future work.

## Literature Review

Stückelberger and Grasshoff (2006) provide the most complete and accurate modern translation of Ptolemy's *Geography* but, unfortunately for us, it is only available in German[3]. Since none of the four co-authors speak fluent German, this made it difficult to use this work other than as a source for names and coordinates from the catalog. However, this turned out to be sufficient for us to make substantial progress. Furthermore, once we learned that Stückelberger and Grasshoff provide an easily accessible database on the CD accompanying their book, we were able to greatly accelerate our work by avoiding much of the scanning and parsing tasks that we had originally anticipated. Stückelberger and Grasshoff suggest modern names for many of the places described by Ptolemy, but their coverage is stronger for other regions than for India, and they provide only the modern names, not the modern coordinates.

McCrindle (1927) fills in many of the gaps Stückelberger and Grasshoff leave by providing another source, this time in English, of all of Ptolemy's coordinates focused on India. He also provides suggestions for many additional modern names, along with a description of his sources and rationale for each choice. However, here again only the names are provided, not a comprehensive set of modern coordinates. The source is not available in machine-readable form, so we attempted automated scanning and parsing. Unfortunately, we found that human understanding of the nuances of the data was required to turn McCrindle's descriptions into usable modern coordinates for the places we considered known. We developed tools to help streamline the process and were able to make it through a large portion of McCrindle's work to extract additional known coordinates.

Berggren and Jones (2000) provide an excellent English translation of the first book of Ptolemy's *Geography*, which is important because it covers the theoretical material. This helped us gain deeper insight into some of the rationale and methods Ptolemy used in coming up with his original estimates and develop better understanding of his major errors. Regretfully, this valuable source only provides full coverage of this one book and, unlike McCrindle, does not help us understand much of the specifics about India or the rest of the catalog.

Stevenson (Ptolemy 1991) attempted to provide a complete English translation of Ptolemy's *Geography*, but his translation is known to contain numerous major flaws (Diller 1935). He does however provide a translation of the entire catalog, so while it must be used very cautiously and compared against other sources, it is still somewhat useful. Stevenson's work is like McCrindle's for us in that it also requires digital extraction efforts.

There have been several noteworthy attempts at the reconstruction of different regions of the an-

[3] It also contains the original Ancient Greek text from the source manuscripts.

cient world based on the data from Ptolemy's Geography.

Strang (1998) divided Ptolemy's points for Britain into groups according to two longitudinal scales and several spatially non-intersecting rotation groups in order to account for the turning of Scotland and other distortions observed in that region. The modern map contours were then warped to superimpose them over Ptolemy's points, thus producing an approximate reconstruction of Ptolemy's map of Britain in Ptolemy's own projection. In our opinion, this kind of reconstruction is less illuminating than those that remap Ptolemy's points into modern projections.

Lacroix (1998) applied conventional linguistic and toponymic analysis of Ptolemaic maps to the difficult task of reconstructing all of Ptolemy's Africa, with limited success.

Berggren and Jones (2000) presented a nearly complete reconstruction of Ptolemy's Gallia (Celtogalatia) by means of explicitly listing most modern locations corresponding to the ancient ones. Unfortunately, this reconstruction was not visualized.

Widespread general recognition of the need for "a rigorous revisiting of Ptolemy's representations, especially the regional tabulae, in terms of georeferencing" (Livieratos 2006) led to publication of important works that dealt with Ptolemy's data, including a paper on Ptolemy's Crete (Livieratos 2006a).

Manoledakis and Livieratos (2007) used Ptolemy's data to determine the approximate location of Aegae, an ancient capital of Macedonia. Their technique of approximate localization is based on transplanting Ptolemy's azimuths into the modern coordinate system and further adjusting them as needed.

Tsorlini (2011) provides a thorough catalogue of Ptolemy's Mediterranean and Black Sea region and a methodology deriving modern coordinates. Since India is not included, the catalogue was not usable for our work here. However, in our future work we hope to compare methodologies for deriving modern coordinates.

From the more general historical perspective of application of mathematical methods to similar problems, it is worth noting that regression analysis (Draper and Smith 1998) has been applied to old maps since Tobler (1966) derived equations to relate the medieval Hereford map to an oblique Mercator projection. While providing a review of other publications devoted to the mathematical analysis of ancient maps is beyond the scope of this paper, we additionally refer the reader to (Ravenhill and Gilg 1974), (Plewe 2003), (Tsotsos and Savvaidis 2003), and (Izaksen 2011).

## Tools and Workflow

We developed a number of tools and techniques in this work that may be useful to other researchers. This section describes these tools and associated workflow organized as five distinct functional areas: scanning, data import, KML generation, geocoding, and visualization.

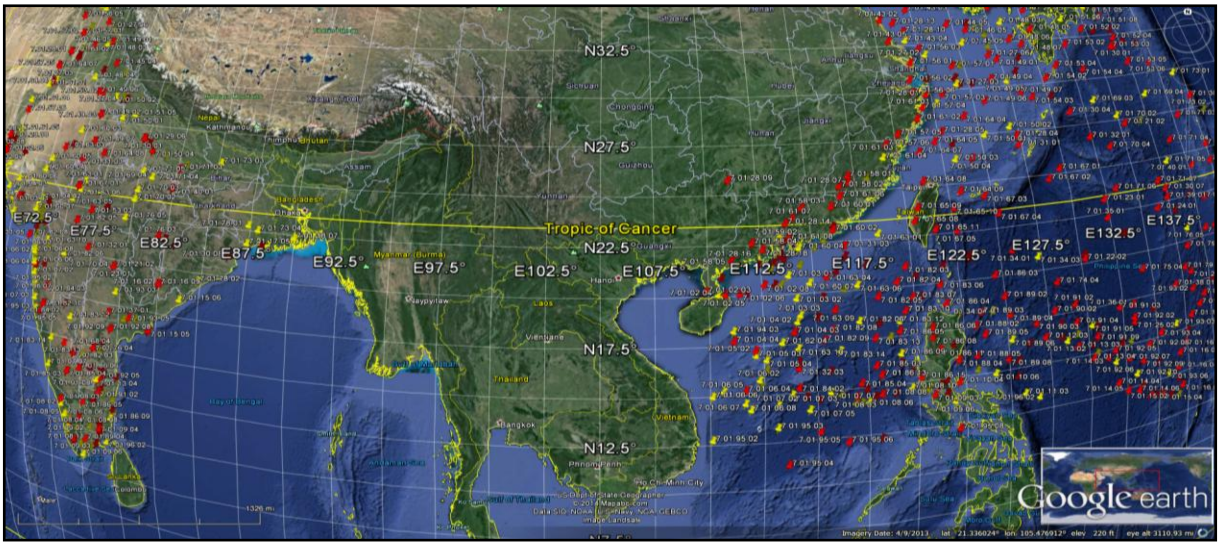


Figure 4. This figure shows a screenshot of Google Earth with the triangulation output KML loaded. We load the Ptolemy coordinates as they are, even though the coordinate system is wrong (especially the prime meridian). It's wrong in a way that is useful for visualization, because it is visible in the same frame as the modern coordinates. In this frame, we can see the modern known and unknown locations on the left over the real India, and the Ptolemy coordinates to the right over the Pacific Ocean. We found it most useful for debugging and further point identification to label the points consistently with the ID system used in Stückelberger and Grasshoff.

### *Scanning*

Given the data-intensive nature of our problem, one challenge we faced was in scanning and parsing the various source texts we needed to use. To this end, we developed automated workflows based on Tesseract (Smith 2007) and ABBYY FineReader (ABBYY 2015), scanners and scanner automation libraries, and custom parsers to extract data tables from raw recognized text. While this added some value early on in our process, we eventually determined that the machine-readable database included by Stückelberger and Grasshoff was sufficient for our initial work. We need future improvements in this area, as there are tables and other data in source texts such as McCrindle that we would like to incorporate into our algorithms and make available for easy reference in our output and visualization tools.

### *Data Import*

We developed software to read the data on the Stückelberger and Grasshoff CD into our algorithms. Stückelberger and Grasshoff provide four main data files: places, categories, people, and realities; however, so far we only need places and, to some degree, categories for our work. Translation remains a challenge for us here. Like the book, the data on the CD is all in German. Even as we would prefer it to be in English, we recognize that for any language we choose for our output many members of our international audience would face a similar problem. Therefore, in addition to translating the data from German to English, we also intend to make our results available in as many other languages as possible. Other internationalization issues, such as determining the correct file encodings for reading, were worked through somewhat painfully and taught us to take special precautions as we move towards publishing our data.

*KML Generation*

Since one of our stated goals with this research is making Ptolemy's work available in Google Earth and related tools, we developed several routines to help us produce KML files from the data. In addition to needing these files as deliverables for our project, we found them invaluable for our own research tasks including identification of known locations, validation of scanning and parsing output, and better understanding and verification of mathematical models. An example of its use is shown in Figure 4. One of the primary libraries we used during our work was simplekml (Lancaster, 2014), which worked well for us initially, but created challenges later on. We intend to create a custom KML library specifically tailored to our purposes. This will enable a significantly enhanced workflow, allowing us to incorporate a faster and more intuitive edit loop on manual known point adjustments and inputs from other researchers interested in this area.

*Geocoding*

As our sources mentioned only the names or descriptions for their modern suggestions for places described by Ptolemy as opposed to the actual modern coordinates, we needed another toolset to help us convert those names into coordinates that we could use to feed our algorithms. We developed two such tools. The first was a program to take in an ID[4] and a place name, interface with the Google Geocoding API (Google, 2015) for the actual geocoding, and output a file containing modern coordinates for that place name that could be referenced during later processing for that ID. The second was a program that generated the same files, but did so by providing us with a minimal GUI to allow us to manually find the locations on a map and copy them over or input them by hand. Both tools could still use some improvements, but saved us countless hours in collecting the data to feed our models.

*Visualization*

Google Earth was a key focus of our effort and became a primary visualization tool for us. The KML generation and Google Earth import steps we built into our workflow were essential to our rapid progress. In addition, three other visualization tools we used are also worth mentioning. The first one is ArcGIS 10.3 that we used to produce maps for the publication. The second is Google Maps because of its usefulness in helping determine where the harder-to-find suggested modern place names might actually be located and looking up their specific coordinates manually. This included tasks such as tracing rivers from mouth to source to match the reference points mentioned by Ptolemy. The third is a custom animation that interpolated all points from their Ptolemy location to their modern equivalents.

[4] We adopted Stückelberger and Grasshoff's Ptolemy ID labeling method.

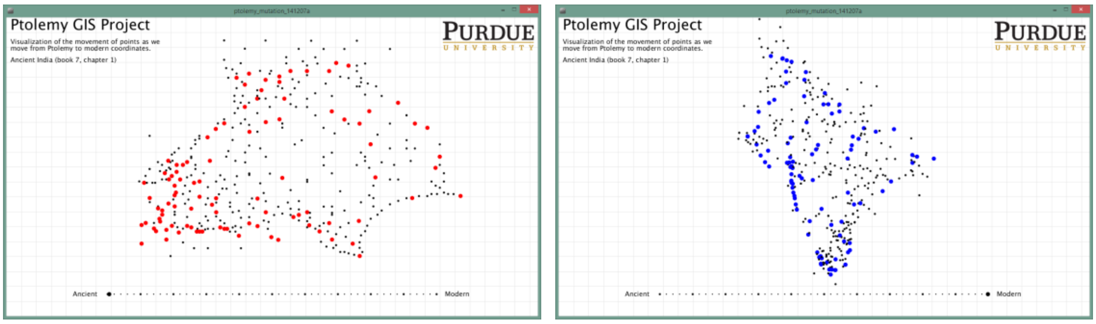


Figure 5. Shown above are the screenshots of the animation visualization sketch in its extreme states. The tool cycles smoothly between ancient and modern coordinates, allowing the eye to follow both the known and unknown points as they move between their two locations. Users can click on the sketch to take manual control of the time bar.

Visualization became essential in helping us understand our models, identify outliers and errors in our geocoding process, and envision new models. A screenshot of our visualization program in action is shown in Figure 5.

## Places

Ptolemy's Geography is a unique ancient work, and identifying the modern equivalent of even a single place mentioned in its catalog can be extremely difficult sometimes. Even for known places, there are often several reasonable modern candidates available in the literature, each with its compelling rationale, and with alternatives being hundreds of kilometers apart. In this section, we provide more information about Ptolemy's Geography, the data that we used to seed our set of known modern equivalents, and our efforts to identify additional known data points to use for predicting where the other data points fall.

### *About Ptolemy's Geography*

Ptolemy's Geography is comprised of several books. The first book describes prior work by other scholars of his time and his improvements to that work along with his own novel contributions. Book 2 begins the catalog part and each subsequent book up to and including Book 7 focuses on a different area of the known world at that time. Because our focus is on India, we primarily investigated Book 7 for the India portion of the catalog, and Book 1 for theoretical underpinnings.

Book 7 is comprised of four chapters, each pertaining to a different region of southern Asia. Chapter 1 is by far the largest and focuses on the Indian subcontinent spanning from modern Pakistan, including the area around the Indus River, to all around the coastline of India, and along Ganges River and the Himalayas to where the Ganges enters the sea. Chapter 2 describes the area beyond the Ganges. Chapter 3 describes the areas located even further east than India. Finally, Chapter 4 describes the island of Taprobane, which is known today as Sri Lanka, former Ceylon. Of these, for our purposes in focusing on modern India, Chapters 1 and 4 are the most important.

Within each chapter, Ptolemy follows a consistent pattern to enumerate all the places. First, he outlines the entire coastal area. Then he lists all the mountain ranges, followed by the sources, major confluences, bends, and mouths of the major rivers. He then proceeds to list the various people of the land along with their major towns. Finally he lists the surrounding islands.

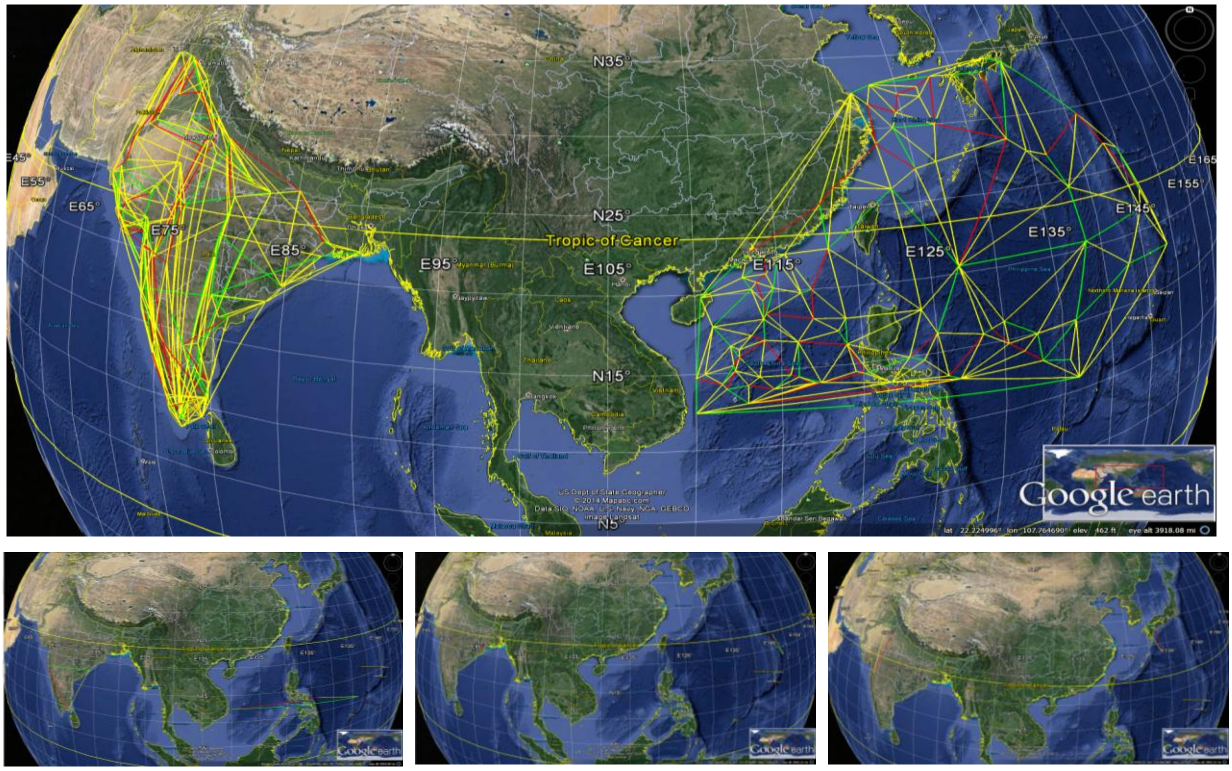


Figure 6. This figure shows the triangulation visualization, depicting especially clearly the Delaunay triangulation used in selecting which known points for each unknown points would be used for the estimation. The colors are inconsistent when viewing them all at the same time as in the top figure, but become clear when only a single triangle is viewed at a time as shown in the bottom figures.

*Stückelberger and Grasshoff's Database*

In the database that accompanies their translation of Ptolemy's work, Stückelberger and Grasshoff list 12,883 unique records in their places table. Of those, 1,217 records pertain to Book 7. Within that set, 640 records actually have a Ptolemy latitude and longitude associated with them. Because of the nature of our work, we filtered out all records that lacked coordinates. Of those that remained, 47 were duplicates by their ID, most of which were there because they represented either an alternate name, or a larger feature such as a mountain, and each row specified a different point within the feature. This leaves 593 unique places in Book 7. Stückelberger and Grasshoff suggest a modern name for 99 of those places, 84 of which are for Chapter 1. During our first pass, we were only able to successfully geocode about 50 of those locations using our program that leveraged Google's geocoding API. This was the initial set we took for further processing to try to derive the other unknown points.

*Additional Points*

After working with the various models for some time, we realized that we really needed additional known points. Using a combination of translations of Stückelberger and Grasshoff names along with McCrindle, Wikipedia, Google searches, and Google Maps, we were able to come up with potential names and coordinates for 85 more places out of the 593 that we would like to be able to plot. The 98 known points for Chapter 1 are listed in Table 1, and the 21 points known for Chapter 4 are listed in Table 2.

## Models

While we know some of the modern equivalents of places that Ptolemy describes, most of them are simply unknown. The challenge we address with our work is to use the few places whose locations are known based on evidence accumulated through the literature to estimate the locations of many that remain unknown. This section describes the models we used to make such estimates.

### *Linear Regression Model*

Like Gusev, Stafeyev and Filatova's (2005) in their work on Ptolemy's Africa, one of our models was a simple linear regression model. We use both ancient latitude and longitude to predict the modern latitude and then separately use the same input data to predict the modern longitude. We used the scikit-learn library for Python (Pedregosa 2011) in our implementation[5].
The model used is the one described earlier in Gusev, Stafeyev and Filatova:

$$\lambda_m = a_0 + a_1\lambda_p + a_2\varphi_p,$$
$$\varphi_m = b_0 + b_1\lambda_p + b_2\varphi_p,$$

where $\lambda_m$ and $\varphi_m$ represent the modern longitude and latitude, $\lambda_p$ and $\varphi_p$ represent the longitude and latitude mentioned by Ptolemy, and the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are the regression coefficients.

### *Triangulation Model*

Also originating from Gusev, Stafeyev and Filatova, this method uses three Ptolemy points for which we know their modern coordinates to form a spherical triangle surrounding a point to be predicted, and then triangulate to find the unknown point. That is, we estimate the unknown modern coordinate pair $\lambda_m$ and $\varphi_m$ using the formulas

$$\lambda_m = \sum_{i=1}^{3} \frac{\lambda_i S_i}{S_1 + S_2 + S_3},$$

$$\varphi_m = \sum_{i=1}^{3} \frac{\varphi_i S_i}{S_1 + S_2 + S_3},$$

maintaining the notation used earlier, and extending it with $\lambda_i$ and $\varphi_i$ as the longitude and latitude of the modern coordinates for the three surrounding points, and $S_i$ as the surface area for the spherical sub-triangle across from the unknown point, which is formed by trisecting the outer triangle by the lines leading from each of its three vertices to the interior unknown point. Like Gusev, Stafeyev and Filatova, we also compute the area of the spherical triangle $S$ mentioned above as

$$S = 4 \arctan \sqrt{\tan\left(\frac{s}{2}\right)\tan\left(\frac{s-a}{2}\right)\tan\left(\frac{s-b}{2}\right)\tan\left(\frac{s-c}{2}\right)}$$

where $a$, $b$, and $c$ are the lengths of the sides of the triangle in radians, and $s = (a + b + c)/2$.

[5] We use aspects of scikit-learn for other models and tasks in our work as well, but do not proceed to exhaustively enumerate them all here.

Finally, again following Gusev, Stafeyev and Filatova, we use the *modified great circle distance* to compute the lengths of the sides of the spherical triangle according to the formula

$$d_{1,2} = 2\arcsin\left[\min\left\{\left(\sin\frac{|\varphi_1-\varphi_2|}{2}\right)^2 + \cos\varphi_1\cos\varphi_2\left(\sin\frac{\gamma|\lambda_1-\lambda_2|}{2}\right)^2\right\}\right],$$

where, like before $\lambda_i$, and $\varphi_i$ represent the longitude and latitude of the two points, and a *narrowing coefficient* of $\gamma$ is applied to account for the local longitudinal stretch of Ptolemy coordinates. Note the constraint that each of the unknown places to be predicted must be enclosed by a spherical triangle of other points that we do know. We found that many unknown locations do not satisfy that criterion, so this model fails to estimate modern coordinates for a significant portion of the catalog. Furthermore, this constraint makes it quite difficult to test and validate the model, because many of the known points that we'd want to validate are on the convex hull, so removing them makes it impossible to estimate their modern locations, and thereby impossible to measure predictive accuracy for them.

However, for the remaining points, and after substantial manual effort to find additional known points along the convex hull of the dataset, this approach turned out to be the most accurate of those we have researched, and seemed the most conceptually straightforward.

Another noteworthy challenge not addressed by Gusev, Stafeyev and Filatova was how to assign the set of unknown points to the sets of points representing their respective surrounding spherical triangles. To address this challenge in an efficient way we computed a Delaunay (1934) triangulation of the known points in their ancient coordinates and looked up the surrounding points for a point to be predicted by querying the results.

*Basis Vector Model*

This model attempts to relax the surrounding triangle constraint, but the price appears to be highly variant results. We find the three nearest known neighbors for each location to be predicted based on their distance to the unknown and then treat them as a basis vector as mentioned by Strang (2009) of the unknown in ancient coordinate space. We use these to construct a matrix representing the basis

$$\begin{bmatrix}\lambda_3-\lambda_1 & \lambda_2-\lambda_1\\ \varphi_3-\varphi_1 & \varphi_2-\varphi_1\end{bmatrix}.$$

We then take the Ptolemy coordinates for the unknown point, $\lambda_4$ and $\varphi_4$ to form a vector $\boldsymbol{b}$ as

$$\begin{bmatrix}\lambda_4-\lambda_1\\ \varphi_4-\varphi_1\end{bmatrix}.$$

We use these to solve for a vector $\boldsymbol{x}$ in

$$A\boldsymbol{x} = \boldsymbol{b}$$

representing the unknown point in terms of the basis formed by the known points.

We form a second basis and the associated matrix $B$ using our modern coordinates for the known points and solve

$$B\boldsymbol{x} = \boldsymbol{c}$$

for the vector $\boldsymbol{c}$, which represents the modern equivalent of the unknown location in terms of the modern basis. We compute the estimated modern coordinates for the unknown point as

$$\begin{bmatrix}\lambda_m\\ \varphi_m\end{bmatrix} = \begin{bmatrix}\lambda_1\\ \varphi_1\end{bmatrix} + \begin{bmatrix}c_\lambda\\ c_\varphi\end{bmatrix}.$$

*Bayesian Adjustment*

The technique developed in this section is not a model on its own. Rather, it takes the output of any of the other models and adjusts it to account for certain prior beliefs, such as that places described by Ptolemy as situated on mainland should fall somewhere on the land mass representing India.

Specifically, we create an image representing the map of India that is black and white, with black representing areas of zero probability and white representing areas of a uniformly distributed probability over the entire subcontinent. This map is loaded as a grid approximation of the probability, normalized so that the entire image (i.e., the grid of probabilities) sums to one. We then take each of the output points and create a second grid probability approximation that treats the output point as the mean of a bivariate normal distribution. We use what Kruschke (2011) shows us about how to infer a binomial proportion via grid approximation to apply Bayes rule

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)},$$

combining the prior with the data and normalizing to arrive at a posterior belief for the new point. For us, $P(A|B)$ is the posterior grid we are interested in, containing the probability distribution of our belief of the new location of the point. The prior $P(A)$ is our belief in where the point must lie without regard to our new data, which for us is the land mass of the Indian subcontinent as represented by the black and white map of India. The new data $P(B|A)$ we take as the output from our input model, a bivariate normal grid representation. The evidence $P(B)$ given our grid approach is the value the causes our grid to be a probability distribution by summing to 1, which is the sum of the entire grid.

We then take the maximum a-posteriori (MAP) of the resulting grid approximation posterior as the new output point. We deemed this approach superior to the maximum likelihood estimator (MLE) for our purposes, because with the latter we still occasionally end up with points in the middle of the ocean. The MAP does not suffer from this, because the point has to have some probability to survive as the MAP. But the points in the ocean are treated as zero probability in our prior, so they have zero probability in the posterior as well. The prior, along with the data and posterior as interim images are shown in Figure 8. We intend to extend this approach to apply our beliefs around other features such as rivers, lakes, and mountains to our other models. For instance, we could similarly load a prior with a grid approximation of a river such as the Ganges to help adjust points describing towns that are described by Ptolemy as near it. The nature of Ptolemy's descriptions and approach will lend itself well to this adjustment.


Figure 7. This figure shows the nearest neighbors for each known point, given another graph similar to the triangulation one shown in Figure 6. As in Figure 6, the colors are more useful when viewing only one point at a time, and folders are provided within the KML to easily turn on and off entire sets to make this more useful. Several of the models we applied use nearest neighbors rather than the surrounding triangle from the Delaunay triangulation, and this visualization proved quite useful in debugging them and in locating new points.

### *Flocking Model*

This attempt was inspired by the animation visualization shown in Figure 5 and described earlier. While watching the ancient to modern point movement of the triangulation model, it was interesting to see how the unknown points moved in relation to their nearest neighbors. Figure 7 gives a visualization of the nearest neighbors, analogous to the earlier visualization of the Delaunay triangulation used for the triangulation approach. This inspired the idea to take a weighted average of the movements of the neighbors, as opposed to trying to average their positions, in some respects similar to the flocking algorithm as described by Reynolds (2001).

Like in the visualization, we first move the center of mass of the entire Ptolemy data set directly over the center of the modern point set, using

$$\lambda'_{p_i} = \lambda_{p_i} - \left( \frac{\sum_{j=1}^{n} \lambda_{p_j}}{n} - \frac{\sum_{j=1}^{n} \lambda_{m_j}}{n} \right),$$

$$\varphi'_{p_i} = \varphi_{p_i} - \left( \frac{\sum_{j=1}^{n} \varphi_{p_j}}{n} - \frac{\sum_{j=1}^{n} \varphi_{m_j}}{n} \right),$$

where $\lambda_{p_i}$ is the $i^{\text{th}}$-place Ptolemy longitude, $\varphi_{p_i}$ is the corresponding Ptolemy latitude, $\lambda_{m_i}$ and $\varphi_{m_i}$ are their respective modern equivalents, and $n$ is the total number of places. Then for each of the known points we compute the vector by which the Ptolemy coordinate must move to become the modern coordinate. That is,

$$\boldsymbol{v}_i = \boldsymbol{m}_i - \boldsymbol{p}_i,$$

where $\boldsymbol{v}_i$ is the movement vector, $\boldsymbol{p}_i$ is the Ptolemy vector computed above, and $\boldsymbol{m}_i$ is the modern coordinate vector corresponding to those Ptolemy vectors.

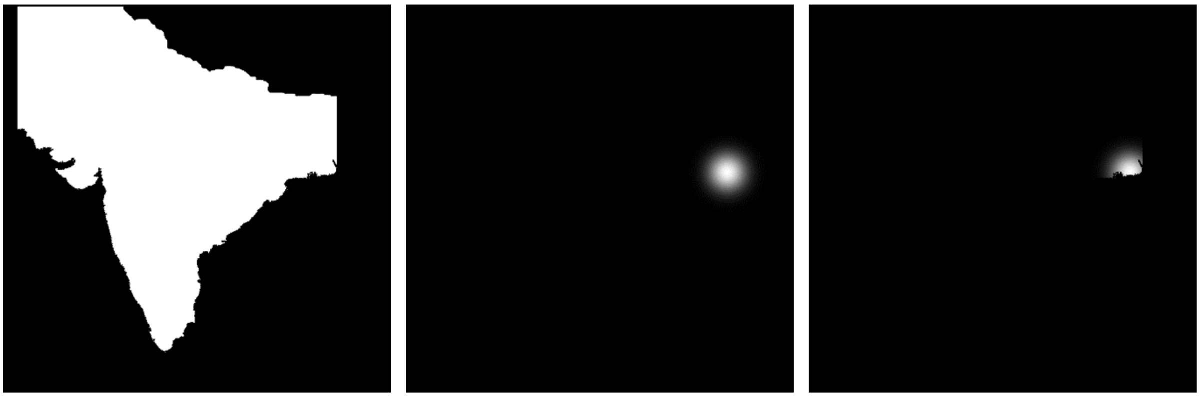

Figure 8. The leftmost figure shows the prior we used for India (book 7, chapter 1). The other figures show the rest of the Bayesian calculation. The prior is on the left, the data is in the center, and the posterior is on the right. We take the MAP of the posterior as the adjusted point.

For prediction, we take the $k$ nearest neighbors of the unknown point $y_p$, use their respective distances to compute weighted average of the movement, and use the obtained average to move the unknown point so that it becomes its modern point

$$y_m = y_p + \sum_{i=1}^{k} v_i w_i$$

where y is the predicted point vector $\lambda_m$ and $\varphi_m$, $v_i$ is the difference of the $i^{\text{th}}$ nearest neighbor of $y$'s $k$ neighbors of its modern coordinate to its Ptolemy coordinate, and $w_i$ is the weight for the $i^{\text{th}}$ neighbor. The weight for each neighbor is computed as

$$w_i = \frac{d_i}{\sum_{j=1}^{k} d_j}.$$

*Unsuccessful Models*

Two additional attempts were made at breaking free of the triangulation constraint. The first of the resulting methods we called our *multilateration approach*. It sought to adopt techniques used by modern GPS technologies, locating a point based on its relative spherical distance from 3 other known points. We constructed spheres based on the modern coordinates and found their intersection, using an SVD based method we found on Stack Overflow (zerm 2011). We do not elaborate further, as we found the overall approach complex and ineffective. However, as we later identified several important flaws in our attempt, we may revisit it later.

We called our second method the *tri-area approach*. We intended to enhance the triangulation model by removing the constraint that the unknown points had to be fully enclosed. The idea was based on what turned out to be a misunderstanding about the way the triangulation approach actually works. We mistakenly thought that after applying the weights to compute the new points, the ratios of the areas of the triangles would be preserved in the new configuration. Unfortunately, this is not true; applying the weighted average does not preserve the ratios of the areas. While we were successful in creating a solution that does seem to preserve the areas of the triangles, it does not appear to match up at all with the previous triangulation approach. We also found this approach ineffective and do not consider it further here.

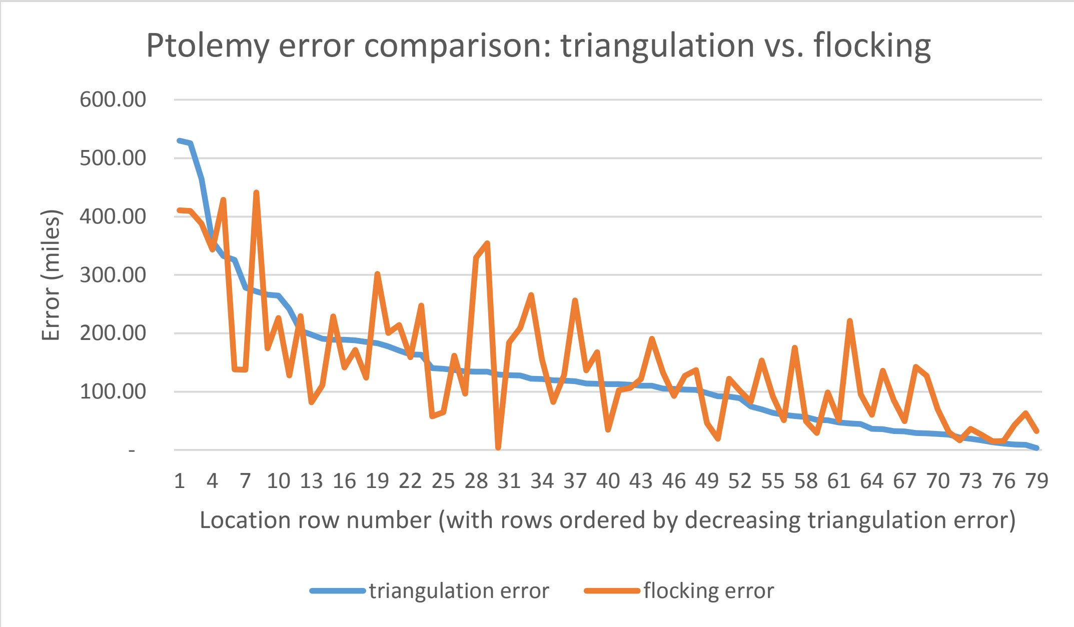


Figure 9. This figure shows the error computed for leave-one-out validation for our two strongest mathematical models: triangulation and flocking. The data has been sorted in decreasing order of the triangulation error. We can see here that the flocking error seems to follow roughly along with the triangulation error, but that there is a high degree of variance along the line, indicating that for some points flocking does better, while for others triangulation does better.

## Results

### *Google Earth*

This was our primary output, especially given how much we made use of it. We found KML to be remarkably powerful, despite its simplicity, in communicating our visualization needs to Google Earth and found the tool to support our workflow well in trying to determine new points to consider as known.

In addition to visualizing the points, we also found it incredibly useful to visualize other geometric artifacts from our models. For instance, the triangulation approach relies heavily on the Delaunay triangulation. Visualization of this triangulation, along with the points that comprise it and the points that fall within each triangle, proved to be quite useful in improving both the model and the data. An example of this visualization is shown in Figure 6.

### *Processing Visualization*

This was a useful visualization for understanding which points moved where, and how the movements compared with one another. It was the inspiration for the flocking model. This tool was developed as a Processing sketch (Raes 2007). Screenshots of the visualization in its two extreme states are given in Figure 5.

### *Error Analysis*

We conducted leave-one-out validation and cross-validation on the two models we found to be most accurate, after applying our Bayesian adjustment to each, for Book 7 Chapter 1, which covers most of modern India, which Ptolemy describes as India before the Ganges. We were only able to compare for regions that were not excluded by the triangulation constraint described earlier in this paper. Our average error for the flocking approach was 145 miles, and our average triangulation error was 132 miles. Figure 9 allows us to visualize how the two models are each more accurate for some points than others. That is, because the plot is sorted by decreasing error on the triangulation approach, the high degree of change shown in the series for the flocking approach means that it was less accurate than triangulation in some cases and more accurate in others. Furthermore, we can also see that as this happens, the two do follow the same trend, with overall error decreasing for flocking as it decreases for triangulation.

## Conclusions and Future Work

Our hope is that this work will stimulate future research interest in this area and serve as a useful foundation for such work. In the rest of this section, we give some recommendations for future projects in this area.

The first and most obvious extension is to simply apply the same concepts and techniques to each of the other books and chapters in Ptolemy's Geography. We focused on India to get started, but the same principles and techniques should work just as well for any of the other regions. In fact, it is likely that other regions may have far better results, because Ptolemy knew those areas better and greater percentages of Ptolemaic places may turn out to be known.

The next extension is to further improve on the known locations within India. We recognize that there is still a degree of uncertainty in respect of many of the places we are classifying as known, and additional work in this area could reduce that amount of uncertainty. A dream scenario would be for archaeologists to travel to the coordinates we provide and find a lost ancient city mentioned by Ptolemy.

Also, we are not doing anything yet to effectively capture the degree to which we consider each place known, while clearly we know some locations with a higher degree of certainty than others. Adopting a rating-like discrete classification of the degree to which each point is known could be useful. We could even go further and describe a full prior distribution for each known, fully capturing our beliefs about its certainty. We already use this concept in our Bayesian adjustment, but it could be utilized to a much greater degree in future work.

We also recognize that the models we developed leave ample opportunity for improvements along several dimensions. For example, we could substantially extend the amount of data we use. The only data we are using in terms of features for prediction are the Ptolemy latitude and longitude. It is worth exploring other potential feature data such as toponym information, tribe names, metadata such as more detailed category information, and further geological feature information. For instance, for mountain identification it may be possible to make use of elevation data to predict more likely coordinates for mountain ranges. Similarly, vector data for river paths could potentially be used to better locate various river-related features, towns and other places that are described in terms of their proximity to such rivers. Other dimensions might include type of model used and applying combinations of models.

We anticipate that many of the tools and techniques described in this paper would be useful in understanding other ancient authors. Indeed, we intend to carry the work through the rest of Ptolemy's *Geography*, providing a complete modern rendition of his *oikoumene* in tools like Google Earth, Google Maps, and ArcGIS.

The source code for the tools developed in this work is available on GitHub[6].

[6] https://github.com/coreyabshire/ptolemy

**Acknowledgments**

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# Tables

| *Ptolemy ID* | *Ptolemy Name* | *Modern Name* | *Ptol. Lat.* | *Ptol. Lon.* | *Mod. Lat.* | *Mod. Lon.* |
|---|---|---|---|---|---|---|
| 7.01.02.03 | Naustathmos | Karachi harbor | 20.00 | 109.75 | 24.85 | 66.98 |
| 7.01.02.04 | Sagapa | Ghizri creek | 19.83 | 110.33 | 24.75 | 67.10 |
| 7.01.03.01 | Bardaxema | Bhadreshwar | 20.67 | 113.67 | 21.64 | 69.63 |
| 7.01.03.02 | Syrastra | Junagadh | 19.50 | 114.00 | 21.17 | 72.83 |
| 7.01.03.03 | Monoglosson | Mangrol | 18.67 | 114.17 | 21.12 | 70.12 |
| 7.01.04.02 | Mophis R. mouth | Mahi | 18.33 | 114.00 | 22.24 | 72.66 |
| 7.01.05.03 | Narmades R. mouth | Narmada | 16.75 | 112.00 | 21.61 | 72.56 |
| 7.01.05.04 | Nusaripa | Navsari | 16.50 | 112.50 | 20.95 | 72.95 |
| 7.01.05.05 | Pulipula | Sanjan | 16.00 | 112.50 | 20.19 | 72.82 |
| 7.01.06.02 | Suppara | Sopara | 15.50 | 112.17 | 19.42 | 72.80 |
| 7.01.06.03 | Goaris R. mouth | Ulhas River/Vasai creek | 15.17 | 112.25 | 19.32 | 72.80 |
| 7.01.06.05 | Bindas R. mouth | Thane creek | 15.00 | 110.50 | 19.05 | 72.98 |
| 7.01.06.06 | Simylla | Chaul | 14.75 | 110.00 | 18.57 | 72.94 |
| 7.01.06.07 | Balepatna | Dabhol | 14.33 | 111.50 | 17.59 | 73.18 |
| 7.01.06.08 | Hippokura | Goregaon (West) | 14.00 | 111.75 | 19.16 | 72.84 |
| 7.01.07.02 | Mandagora | Mandangarh | 14.17 | 113.00 | 17.98 | 73.25 |
| 7.01.07.03 | Byzantion | Vijayadurg | 14.67 | 113.67 | 16.55 | 73.34 |
| 7.01.07.05 | Nanagouna R. mouth | Tapti River/Hazira creek | 13.83 | 114.50 | 21.07 | 72.68 |
| 7.01.07.07 | Nitra | Mangaluru | 14.67 | 115.50 | 12.91 | 74.84 |
| 7.01.08.02 | Tyndis | Thikkodi | 14.50 | 116.00 | 11.50 | 75.62 |
| 7.01.08.04 | Cape Kalaikarias | Kozhikode | 14.00 | 116.67 | 11.26 | 75.78 |
| 7.01.08.05 | Muziris | Kodungallur | 14.00 | 117.00 | 10.22 | 76.20 |
| 7.01.08.06 | Pseudostomos R. mouth | Periyar R. mouth | 14.00 | 117.33 | 10.18 | 76.16 |
| 7.01.08.10 | Bakare | Pirakkad | 14.50 | 119.50 | 10.06 | 76.46 |
| 7.01.08.11 | Baris R. mouth | Pamba R. mouth | 14.33 | 120.00 | 9.31 | 76.38 |
| 7.01.09.02 | Melkynda | Nirkunnam | 14.33 | 120.33 | 9.41 | 76.35 |
| 7.01.09.06 | Cape Komaria | Cape Comorin | 13.50 | 121.75 | 8.09 | 77.54 |
| 7.01.10.05 | Kolchoi | Korkei | 15.00 | 123.00 | 8.63 | 78.07 |
| 7.01.10.06 | Solen R. mouth | Tamraparni R. mouth | 14.67 | 124.00 | 8.63 | 78.11 |
| 7.01.11.03 | Cape Kalligikon | Point Callimere | 13.33 | 125.67 | 9.29 | 79.31 |
| 7.01.13.02 | Chaberis | Tranquebar | 15.75 | 128.33 | 11.03 | 79.85 |
| 7.01.14.02 | Poduke | Virampatnam | 14.75 | 130.25 | 11.89 | 79.82 |
| 7.01.15.06 | Departure point to Golden Chersones | | 11.00 | 136.33 | 18.16 | 83.78 |
| 7.01.17.05 | Adamas R. mouth | Subarnarekha R. mouth | 18.00 | 142.67 | 21.56 | 87.37 |
| 7.01.18.07 | Antibole R. mouth | Baleshwari R. mouth | 18.25 | 148.50 | 22.07 | 89.94 |
| 7.01.27.01 | Indus R. confluence with Koas | Indus R. conflu-ence with Konar | 31.00 | 124.50 | 33.92 | 72.23 |
| 7.01.27.02 | Koas R. confluence with Suastos | Konar R. conflu-ence with Swat | 31.67 | 122.50 | 34.11 | 71.71 |
| 7.01.27.03 | Indus R. confluence with Zaradros | Indus R. conflu-ence with Sutlej | 30.00 | 124.00 | 29.15 | 70.72 |
| 7.01.27.07 | Bidaspes R. conflu-ence with Sandabal | Jhelum R. conflu-ence with Chinab | 32.67 | 126.67 | 31.17 | 72.15 |
| 7.01.33.02 | Pseudostomos R. bend | Periyar R. bend | 17.25 | 118.50 | 9.58 | 77.11 |
| 7.01.34.03 | Solen R. sources in Bettigo Mtns | Tamraparni R. sources in S. Ghats | 20.50 | 127.00 | 8.69 | 77.36 |
| 7.01.34.04 | Solen R. bend | Tamraparni R. bend | 18.00 | 124.00 | 8.69 | 77.68 |
| 7.01.43.04 | Dionysopolis | near Jalalabad | 32.50 | 121.50 | 34.44 | 70.40 |
| 7.01.44.04 | Poklais | Charsadda | 33.00 | 123.00 | 34.15 | 71.74 |
| 7.01.45.05 | Taxila | Taxila | 32.25 | 125.00 | 33.74 | 72.80 |
| 7.01.46.04 | Euthydemia | Sialkot | 32.00 | 126.67 | 32.49 | 74.53 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 7.01.48.03 | Labokla | Lahore | 33.33 | 128.00 | 31.56 | 74.36 |
| 7.01.48.04 | Batanagra | Hanumangarh | 33.33 | 130.00 | 29.58 | 74.32 |
| 7.01.49.05 | Indabara | Indraprastha | 30.00 | 127.25 | 28.61 | 77.25 |
| 7.01.50.01 | Modura | Mathura | 27.17 | 125.00 | 27.49 | 77.67 |
| 7.01.50.02 | Gagasmira | Jhajjar | 27.50 | 126.67 | 28.61 | 76.66 |
| 7.01.50.03 | Erarassa | Varanasi | 26.00 | 123.00 | 25.32 | 82.98 |
| 7.01.51.04 | Konta | Kunda | 34.33 | 133.50 | 25.72 | 81.52 |
| 7.01.51.05 | Margara | Marehra | 34.00 | 135.00 | 27.74 | 78.57 |
| 7.01.51.06 | Batankaisara | Thanesar | 33.33 | 132.67 | 29.96 | 76.82 |
| 7.01.59.01 | Patala | Hyderabad | 21.00 | 112.83 | 25.39 | 68.37 |
| 7.01.59.02 | Barbarei | Bhambore | 22.50 | 113.25 | 24.75 | 67.52 |
| 7.01.60.03 | Auxoamis | Ajmer | 22.33 | 115.50 | 26.45 | 74.64 |
| 7.01.60.04 | Asinda | Siddhpur, Gujarat | 22.00 | 114.25 | 23.92 | 72.37 |
| 7.01.60.05 | Orbadaru | Mt.Abu | 22.00 | 115.00 | 24.59 | 72.71 |
| 7.01.60.06 | Theophila | Devaliya | 21.17 | 114.25 | 23.03 | 70.00 |
| 7.01.60.07 | Astakapra | Hathab | 20.25 | 114.67 | 21.57 | 72.27 |
| 7.01.61.01 | Panassa | Bhagsar | 29.00 | 122.50 | 28.83 | 70.22 |
| 7.01.61.03 | Naagramma | Naushehra | 27.00 | 120.00 | 32.57 | 72.15 |
| 7.01.61.04 | Kamigara | Sukkur | 26.33 | 119.00 | 27.71 | 68.85 |
| 7.01.61.05 | Binagara | Brahmanabad | 25.33 | 118.00 | 25.88 | 68.78 |
| 7.01.62.04 | Barygaza | Bharuch | 17.33 | 113.25 | 21.71 | 73.00 |
| 7.01.63.01 | Agrinagara | Agar Malwa | 22.50 | 118.25 | 23.71 | 76.01 |
| 7.01.63.05 | Xerogerei | Dhar | 19.83 | 116.33 | 22.60 | 75.30 |
| 7.01.63.06 | Ozene | Ujjain | 20.00 | 117.00 | 23.18 | 75.78 |
| 7.01.63.10 | Nasika | Nasik (Nashik) | 17.00 | 114.00 | 20.00 | 73.79 |
| 7.01.69.03 | Stagabaza | Bhojapur | 28.50 | 133.00 | 19.68 | 74.04 |
| 7.01.69.04 | Bardaotis | Bharhut | 28.50 | 137.50 | 24.45 | 80.88 |
| 7.01.70.02 | Bridama | Bilhari | 27.50 | 134.50 | 23.14 | 79.97 |
| 7.01.70.03 | Tholobana | Bahoriband | 27.00 | 136.33 | 23.67 | 80.07 |
| 7.01.71.05 | Panassa | Panna | 24.50 | 137.67 | 24.72 | 80.18 |
| 7.01.73.01 | Sambalaka | Sambhal | 29.50 | 141.00 | 28.59 | 78.57 |
| 7.01.73.03 | Palimbothra | Patna | 27.00 | 143.00 | 25.61 | 85.14 |
| 7.01.73.04 | Tamalites | Tamluk | 26.50 | 144.50 | 22.30 | 87.92 |
| 7.01.76.05 | Ozoana | Seoni | 20.50 | 138.25 | 22.09 | 79.54 |
| 7.01.78.01 | Kartinaga | Karnigar | 23.00 | 146.00 | 22.51 | 87.36 |
| 7.01.78.02 | Kartasina | Berhampur | 21.67 | 145.50 | 19.31 | 84.79 |
| 7.01.82.06 | Baithana | Paithan | 18.17 | 117.00 | 19.48 | 75.38 |
| 7.01.83.12 | Modogulla | Mudgal | 18.00 | 119.00 | 16.01 | 76.44 |
| 7.01.83.14 | Banauasei | Banavasi | 16.75 | 116.00 | 14.53 | 75.02 |
| 7.01.86.09 | Karura | Tirukkarur | 16.33 | 119.00 | 10.77 | 79.64 |
| 7.01.89.06 | Modura | Madurai | 16.33 | 125.00 | 9.93 | 78.12 |
| 7.01.91.05 | Orthura | Uraiyar | 16.33 | 130.00 | 12.09 | 79.14 |
| 7.01.91.08 | Abur | Ambur | 16.00 | 129.00 | 12.79 | 78.72 |
| 7.01.92.04 | Karige | Kadapa | 15.00 | 132.67 | 14.47 | 78.82 |
| 7.01.92.06 | Pikendaka | Penukonda | 14.00 | 131.50 | 14.08 | 77.60 |
| 7.01.92.10 | Malanga | Eluru | 13.00 | 133.00 | 16.70 | 81.10 |
| 7.01.93.03 | Bardamana | Vada | 15.25 | 136.25 | 17.97 | 79.59 |
| 7.01.93.06 | Pityndra | Dharanikota | 12.50 | 135.50 | 16.56 | 80.34 |
| 7.01.94.03 | Barake | Beyt Dwarka | 18.00 | 111.00 | 22.46 | 69.10 |
| 7.01.95.02 | Milizigeris | Jaygarh | 12.50 | 110.00 | 17.29 | 73.22 |
| 7.01.95.03 | Heptanesia | Vengurla rocks | 13.00 | 113.00 | 15.93 | 73.46 |
| 7.01.96.02 | Kory | Rameswaram | 13.00 | 126.50 | 9.29 | 79.31 |

Table 1. Modern coordinates for known locations in Book 7 Chapter 1.

| *Ptolemy ID* | *Ptolemy Name* | *Modern Name* | *Ptol. Lat.* | *Ptol. Lon.* | *Mod. Lat.* | *Mod. Lon.* |
|---|---|---|---|---|---|---|
| 7.04.02.01 | North Cape | Point Pedro | 12.50 | 126.00 | 9.82 | 80.23 |
| 7.04.03.02 | Cape Galyba | Kovilan Point | 11.33 | 124.00 | 9.76 | 79.86 |
| 7.04.03.05 | Cape Anarismundu | Kudiramalei Point | 7.75 | 122.00 | 8.44 | 79.85 |
| 7.04.03.09 | Priapis Harbor | Negombo Lagoon | 3.67 | 122.00 | 7.19 | 79.86 |
| 7.04.04.03 | Cape of Zeus | Galbokka Point | 1.00 | 120.50 | 6.94 | 79.84 |
| 7.04.04.07 | Odoka | Hikkaduwa | (2.00) | 123.00 | 6.15 | 80.11 |
| 7.04.04.08 | Birds' Cape | Point de Galle | (2.50) | 125.00 | 6.03 | 80.22 |
| 7.04.05.01 | Dagana, a town sacred to the Moon | Dondra | (2.00) | 126.00 | 5.93 | 80.59 |
| 7.04.05.02 | Korkobara | Tangalle | (2.33) | 127.67 | 6.03 | 80.79 |
| 7.04.05.04 | Cape Ketaion | Elephant Rock | (0.67) | 132.50 | 6.36 | 81.47 |
| 7.04.05.08 | Mordula | Arugam Bay | 2.33 | 131.00 | 6.84 | 81.83 |
| 7.04.06.01 | Abaraththa | Akkaraipattu | 3.25 | 131.00 | 7.22 | 81.85 |
| 7.04.06.02 | Helios Harbor | Batticaloa Lagoon | 4.00 | 130.00 | 7.73 | 81.68 |
| 7.04.06.06 | Cape Oxeia | Foul Point | 7.50 | 130.00 | 8.52 | 81.32 |
| 7.04.06.07 | Ganges R. mouth | Mahaweli Ganges R. mouth | 7.33 | 129.00 | 8.46 | 81.23 |
| 7.04.07.01 | Nagadiba | Nagadeepa Rajamaha Vihara | 8.50 | 129.00 | 9.61 | 79.77 |
| 7.04.07.03 | Anubingara | Kuchchaveli | 9.67 | 128.67 | 8.82 | 81.10 |
| 7.04.07.04 | Moduttu | Kokkilai | 11.00 | 128.00 | 9.00 | 80.95 |
| 7.04.07.07 | Talakori | Thondaimanaru | 11.67 | 126.33 | 9.82 | 80.14 |
| 7.04.10.01 | Anurogrammon | Anuradhapura | 8.67 | 124.17 | 8.35 | 80.39 |
| 7.04.10.02 | Maagrammon | Tissamaharama | 7.33 | 127.00 | 6.28 | 81.28 |
| 7.04.12.04 | Kalandadrua | Colombo | (5.50) | 121.00 | 6.92 | 79.85 |

Table 2. Modern coordinates for known locations in Book 7 Chapter 4.